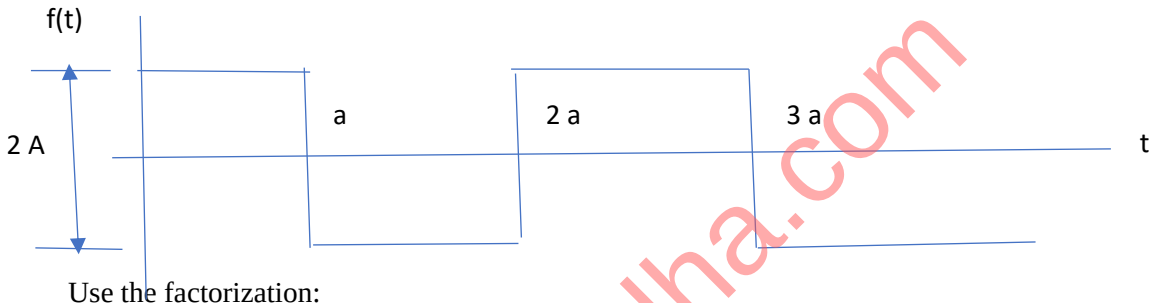
| | |
|---|---|
| Name of Course | : **CBCS B.Sc. (H) Mathematics** |
| Unique Paper Code | : **32357502_LOCF** |
| Name of Paper | : **DSE1-Mathematical Modeling and Graph Theory** |
| Semester | : **V** |
| Duration | : **3 hours** |
| Maximum Marks | : **75 Marks** |

*Attempt any four questions. All questions carry equal marks.*

1. Consider a spring-mass dashpot system with mass m = 1, the viscous damping c = 0 and the spring modulus k = 9 with external force f(t) = 6 cos 3t. Let y(t) denotes the displacement of the mass from its equilibrium position. Find y(t).
   Determine the Laplace Transform of the square-wave function shown in the figure below.




Use the factorization:

$$s^4+4a^4=\left(s^2-2as+2a^2\right)\left(s^2+2as+2a^2\right)$$

and apply inverse Laplace transform to show that:

$$L^{-1}\left\{\frac{1}{s^4+4a^4}\right\}=\frac{1}{4a^3}\left(\cos hat\sin at-\sin hat\cos at\right)$$

2. Find the power series solution of the IVP $y''+9y=0, \qquad y(0)=1, y'(0)=0$.
   What is an Analytic function and a regular singular point? Find the singular points for the ordinary differential equation $x(x-2)^3 y''+2(x-2)^3 y'+3y=0$. .Also check whether the singular points are regular or irregular.

   Find two linearly independent Frobenius series solutions of the differential equation

$$x^2y''-xy'+\left(\frac{3}{4}-x^2\right)y=0$$

3. Consider the following Linear Programming Problem:
   Maximize $Z=6X_1+5X_2$
   Subject to

$$X_1+X_2 \le 5$$
$$3X_1+2X_2 \le 12$$
$$X_1, X_2 \ge 0$$

Solve this Problem using tabular form of Simplex Method.

Explain middle square method. Is there any drawback of this method? Take initial seed $x_0 = 14$ and generate two digit eight random numbers. Is there any problem to generate eleven, two-digit random numbers when seed is $x_0 = 14$.

Write steps of Monte Carlo simulation algorithm to calculate the volume of the following ellipsoid in the first octant:

$$\frac{x^2}{4} + \frac{y^2}{8} + \frac{z^2}{16} \leq 32$$

That lies in the first octant $x > 0,\ y > 0,\ z > 0$

4. Consider the following Linear Programming Problem:
Minimize $Z = X_1 + 3X_2$
Subject to

$$5X_1 + 4X_2 \geq 20$$
$$3X_1 + 4X_2 \leq 24$$
$$X_1, X_2 \geq 0$$

Plot the feasible reason of the problem. Determine the optimal solution from the graph. What would be the optimal solution if the objective function were to be maximized?

Consider the following Linear Programming Problem
Maximize $Z = 3X_1 + 5X_2$
Subject to

$$X_1 + 2X_2 \leq 2000$$
$$X_1 + X_2 \leq 1500$$
$$X_2 \leq 600$$
$$X_1, X_2 \geq 0$$

Determine the sensitivity of the optimal solution to change in $C_1$ using the objective function $C_1x_1 + 5x_2$

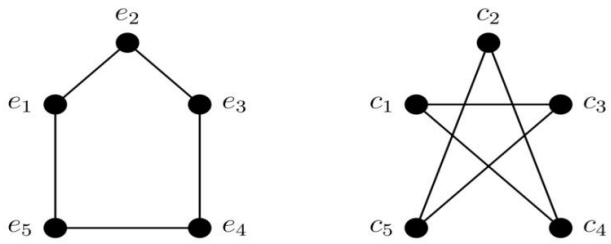
A small harbour has unloading facilities for ships. Only one ship can be unloaded at any one time. The unloading time require for a ship depends on the type and the amount of cargo and varies from 45 to 90 minutes. Below is given a situation with 5 ships:

| | Ship 1 | Ship 2 | Ship 3 | Ship 4 | Ship 5 |
|---|---|---|---|---|---|
| Time between successive ships | 10 | 20 | 5 | 110 | 15 |
| Unload time | 45 | 35 | 50 | 65 | 70 |

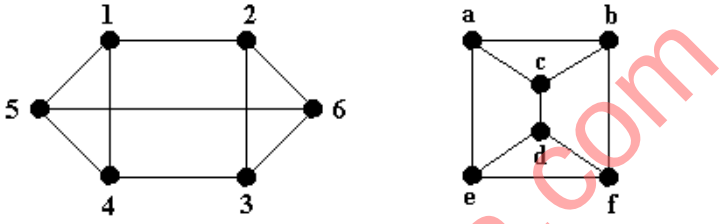
Draw the time-line diagram depicting clearly the situation for each ship, the idle time for harbour and the waiting time. Also list the waiting times for all the ships and find the average waiting time.

5. Verify and explain whether the following graphs are isomorphic:

a)

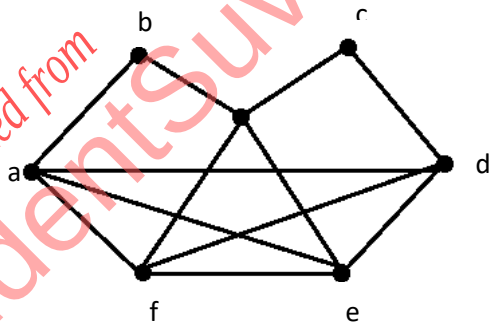



b)




True or false: A graph is bipartite iff every cycle of a graph has even number of edges. Explain?

Can, an Eulerian graph can be split into cycles, no two of which have an edge in common? Justify your answer. Also illustrate the result using the following graph




6. Is a bipartite graph with odd number of vertices not Hamiltonian. Explain? Verify whether the Petersen graph is semi-Hamiltonian or not?
   Show how a Gray code represents a Hamiltonian path in 2-cube and 3-cube.
   For the graph shown below, write down a walk of length 7 between A and I and a cycle of length 3, 4 and 5.